Circuits

by

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Table 1-1 Fundamental and electrical SI units.

Dimension	Unit	Symbol
Fundamental:		
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric charge	coulomb	C
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd
Electrical:		
Current	ampere	A
Voltage	volt	V
Resistance	ohm	Ω
Capacitance	farad	F
Inductance	henry	H
Power	watt	W
Frequency	hertz	Hz

Table 1-2 **Multiple and submultiple prefixes.**

Prefix	Symbol	Magnitude
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Table 1-3 **Symbols for common circuit elements.**

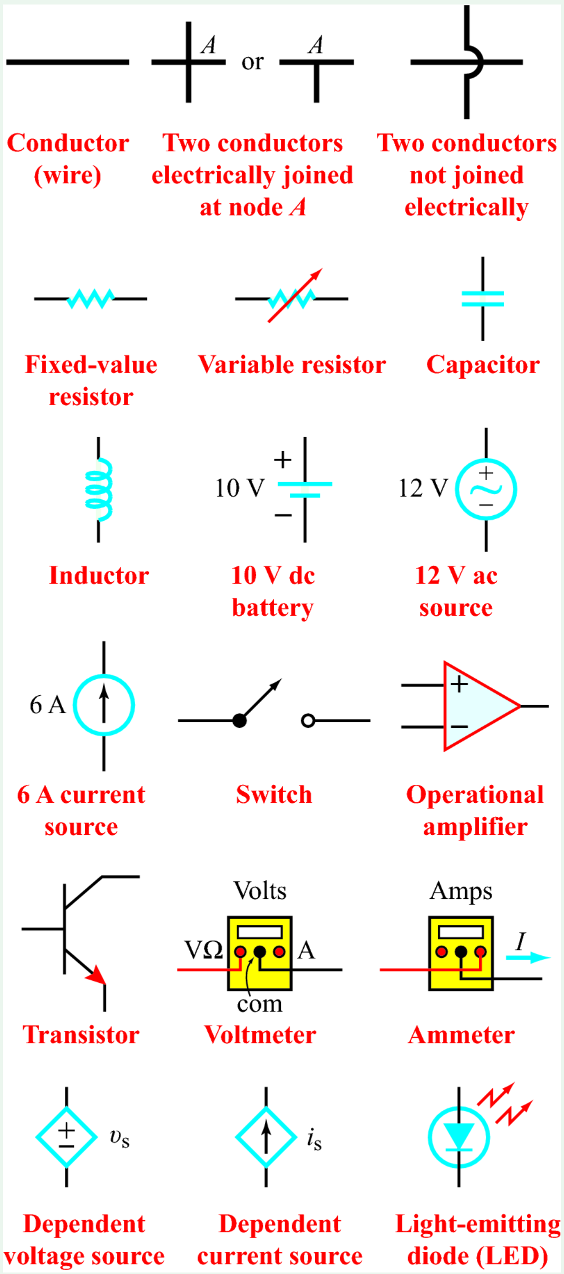


Table 1-4 Circuit terminology.

Node: An electrical connection between two or more elements.

Ordinary node: An electrical connection node that connects to only two elements.

Extraordinary node: An electrical connection node that connects to three or more elements.

Branch: Trace between two consecutive nodes with only one element between them.

Path: Continuous sequence of branches with no node encountered more than once.

Extraordinary path: Path between two adjacent extraordinary nodes.

Loop: Closed path with the same start and end node.

Independent loop: Loop containing one or more branches not contained in any other independent loop.

Mesh: Loop that encloses no other loops.

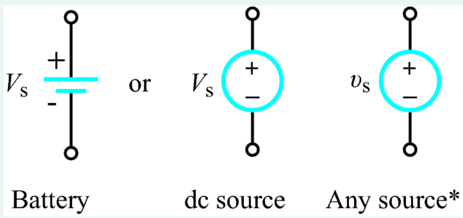
In series: Elements that share the same current. They have only ordinary nodes between them.

In parallel: Elements that share the same voltage. They share two extraordinary nodes.

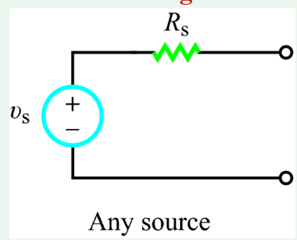
Table 1-5 **Voltage and current sources.**

Independent Sources

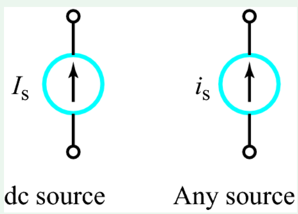
Ideal Voltage Source



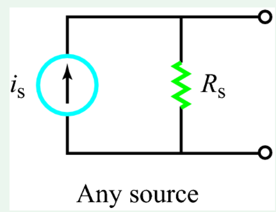
Realistic Voltage Source



Ideal Current Source

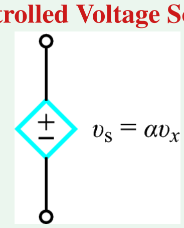


Realistic Current Source

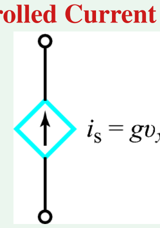


Dependent Sources

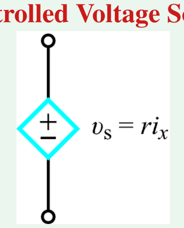
Voltage-Controlled Voltage Source (VCVS)



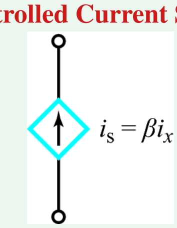
Voltage-Controlled Current Source (VCCS)



Current-Controlled Voltage Source (CCVS)



Current-Controlled Current Source (CCCS)



Note: α, g, r, and β are constants; υ_x and i_x are a specific voltage and a specific current elsewhere in the circuit. *Lowercase υ and i represent voltage and current sources that may or may not be time-varying, whereas uppercase V and I denote dc sources.

Table 2-1 Conductivity and resistivity of some common materials at 20 °C.

Material	Conductivity σ (S/m)	Resistivity ρ (Ω-m)
Conductors		
Silver	6.17×10^{7}	1.62×10^{-8}
Copper	5.81×10^{7}	1.72×10^{-8}
Gold	4.10×10^{7}	2.44×10^{-8}
Aluminum	3.82×10^{7}	2.62×10^{-8}
Iron	1.03×10^{7}	9.71×10^{-8}
Mercury (liquid)	1.04×10^{6}	9.58×10^{-8}
Semiconductors		
Carbon (graphite)	7.14×10^{4}	1.40×10^{-5}
Pure germanium	2.13	0.47
Pure silicon	4.35×10^{-4}	2.30×10^{3}
Insulators		
Paper	$\sim 10^{-10}$	$\sim 10^{10}$
Glass	$\sim 10^{-12}$	$\sim 10^{12}$
Teflon	$\sim 3.3 \times 10^{-13}$	$\sim 3 \times 10^{12}$
Porcelain	$\sim 10^{-14}$	$\sim 10^{14}$
Mica	$\sim 10^{-15}$	$\sim 10^{15}$
Polystyrene	$\sim 10^{-16}$	$\sim 10^{16}$
Fused quartz	$\sim 10^{-17}$	$\sim 10^{17}$
Common materials		
Distilled water	5.5×10^{-6}	1.8×10^{5}
Drinking water	$\sim 5 \times 10^{-3}$	~ 200
Sea water	4.8	0.2
Graphite	1.4×10^{-5}	71.4×10^{3}
Rubber	1×10^{-13}	1×10^{13}
Biological tissues		
Blood	~ 1.5	~ 0.67
Muscle	~ 1.5	~ 0.67
Fat	~ 0.1	10

Table 2-2 **Diameter d of wires, according to the American Wire Gauge (AWG) system.**

AWG Size Designation	Diameter d (mm)
0	8.3
2	6.5
4	5.2
6	4.1
10	2.6
14	1.6
18	1.0
20	0.8

Table 2-3 Common resistor terminology.

Thermistor	R sensitive to temperature
Piezoresistor	R sensitive to pressure
Light-dependent R (LDR)	R sensitive to light intensity
Rheostat	2-terminal variable resistor
Potentiometer	3-terminal variable resistor

Table 2-4 Equally valid, multiple statements of Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL).

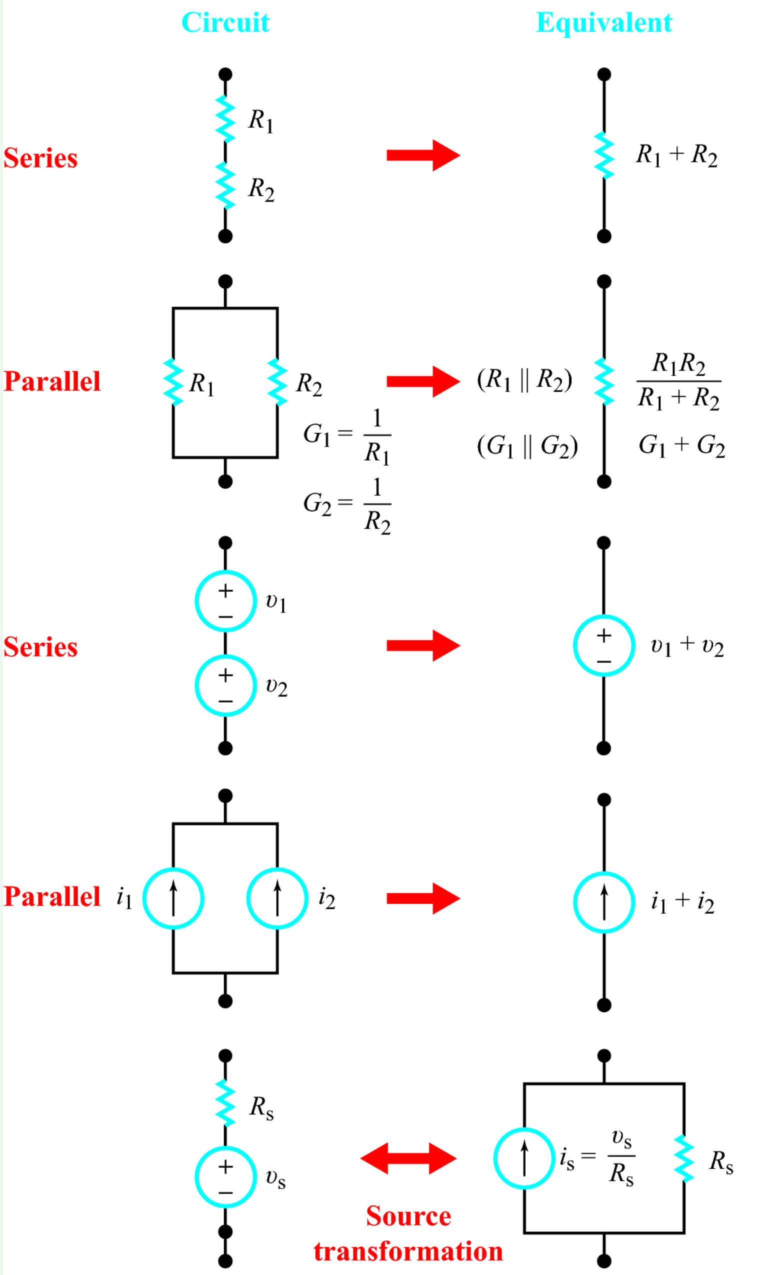
KCL

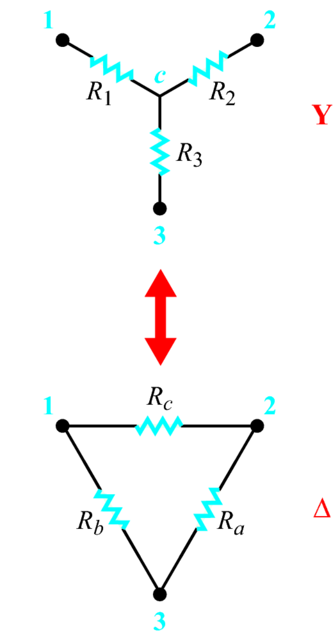
- Sum of all currents entering a node $= 0$
  [$i =$ "+" if entering; $i =$ "−" if leaving]
- Sum of all currents leaving a node $= 0$
  [$i =$ "+" if leaving; $i =$ "−" if entering]
- Total of currents entering $=$ Total of currents leaving

KVL

- Sum of voltages around closed loop $= 0$
  [$\upsilon =$ "+" if + side encountered first in clockwise direction]
- Total voltage rise $=$ Total voltage drop

Table 2-5 Equivalent circuits.





$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

- For $R_a = R_b = R_c \rightarrow R_1 = R_2 = R_3 = R_a/3$
- For $R_1 = R_2 = R_3 \rightarrow R_a = R_b = R_c = 3R_1$

Table 3-1 **Properties of Thévenin/Norton analysis techniques.**

To Determine	Method	Can Circuit Contain Dependent Sources?	Relationship
υ_{Th}	Open-circuit υ	Yes	$\upsilon_{\mathrm{Th}} = \upsilon_{\mathrm{oc}}$
υ_{Th}	Short-circuit i (if R_{Th} is known)	Yes	$\upsilon_{\mathrm{Th}} = R_{\mathrm{Th}} i_{\mathrm{sc}}$
R_{Th}	Open/short	Yes	$R_{\mathrm{Th}} = \upsilon_{\mathrm{oc}}/i_{\mathrm{sc}}$
R_{Th}	Equivalent R	No	$R_{\mathrm{Th}} = R_{\mathrm{eq}}$
R_{Th}	External source	Yes	$R_{\mathrm{Th}} = \upsilon_{\mathrm{ex}}/i_{\mathrm{ex}}$
$i_{\mathrm{N}} = \upsilon_{\mathrm{Th}}/R_{\mathrm{Th}}$; $R_{\mathrm{N}} = R_{\mathrm{Th}}$			

Table 3-2 Summary of circuit analysis methods.

Method	Common Use
Ohm's law	Relates V, I, R. Used with all other methods to convert $V \Leftrightarrow I$.
R, G in series and $\parallel$	Combine to simplify circuits. R in series adds, and is most often used. G in $\parallel$ adds, so may be used when much of the circuit is in parallel.
Y-Δ or Π-T	Convert resistive networks that are not in series or in $\parallel$ into forms that can often be combined in series or in $\parallel$. Also simplifies analysis of bridge circuits.
Voltage/current dividers	Common circuit configurations used for many applications, as well as handy analysis tools. Dividers can also be used as combiners when used "backwards."
Kirchhoff's laws (KVL/KCL)	Solve for branch currents. Often used to derive other methods.
Node-voltage method	Solves for node voltages. Probably the most commonly used method because (1) node voltages are easy to measure, and (2) there are usually fewer nodes than branches and therefore fewer unknowns (smaller matrix) than KVL/KCL.
Mesh-current method	Solves for mesh currents. Fewer unknowns than KVL/KCL, approximately the same number of unknowns as node voltage method. Less commonly used, because mesh currents seem less intuitive, but useful when combining additional blocks in cascade.
Node-voltage by-inspection method	Quick, simplified way of analyzing circuits. Very commonly used for quick analysis in practice. Limited to circuits containing only independent current sources.
Mesh-current by-inspection method	Quick, simplified way of analyzing circuits. Very commonly used for quick analysis in practice. Limited to circuits containing only independent voltage sources.
Superposition	Simplifies circuits with multiple sources. Commonly used for both calculation and measurement.
Source transformation	Simplifies circuits with multiple sources. Commonly used for both calculation/design and measurement/test applications.
Thévenin and Norton equivalents	Very often used to simplify circuits in both calculation and measurement applications. Also used to analyze cascaded systems. Thévenin is the more commonly used form, but Norton is often handy for analyzing parallel circuits. Source transformation allows easy conversion between Thévenin and Norton.
Input/output resistance (R_{in}/R_{out})	Commonly used to evaluate when cascaded circuits can be analyzed individually or when full circuit analysis or a buffer is needed.

Table 4-1 **Characteristics and typical ranges of op-amp parameters. The rightmost column represents the values assumed by the ideal op-amp model.**

Op-Amp Characteristics	Parameter	Typical Range	Ideal Op Amp
• Linear input-output response	Open-loop gain A	10^4 to 10^8 (V/V)	∞
• High input resistance	Input resistance R_i	10^6 to 10^{13} Ω	∞ Ω
• Low output resistance	Output resistance R_o	1 to 100 Ω	0 Ω
• Very high gain	Supply voltage V_cc	5 to 24 V	As specified by manufacturer

Table 4-2 **Characteristics of the ideal op-amp model.**

Ideal Op Amp

- Current constraint $i_p = i_n = 0$
- Voltage constraint $\upsilon_p = \upsilon_n$
- $A = \infty$ $R_i = \infty$ $R_o = 0$

Table 4-3 Summary of op-amp circuits.

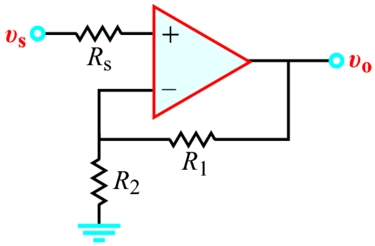
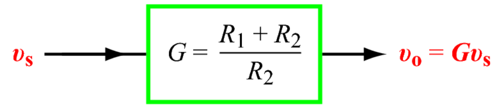
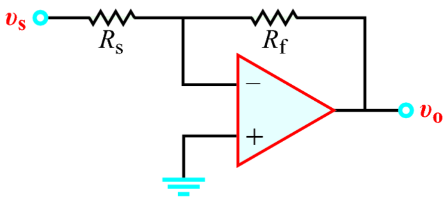
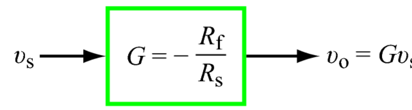
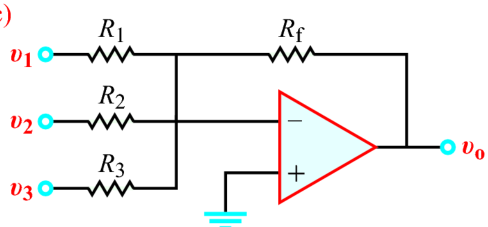
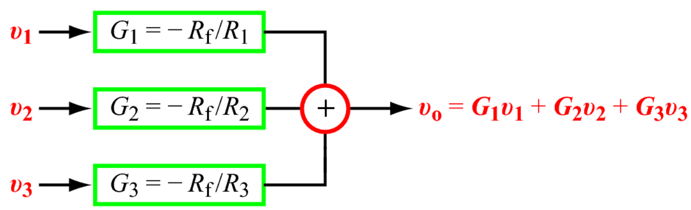
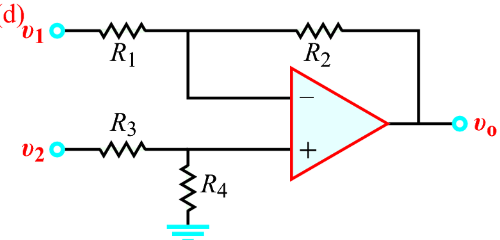
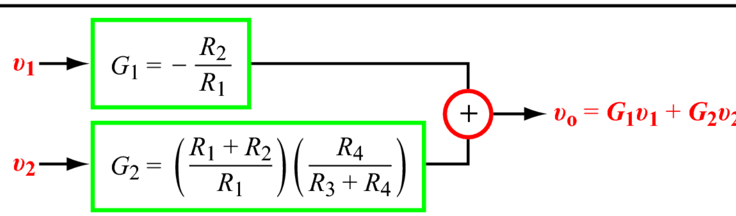
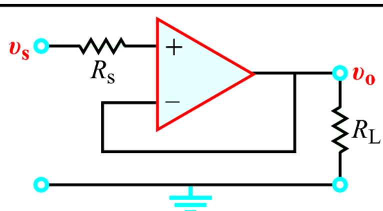
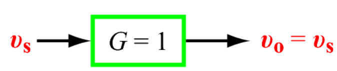
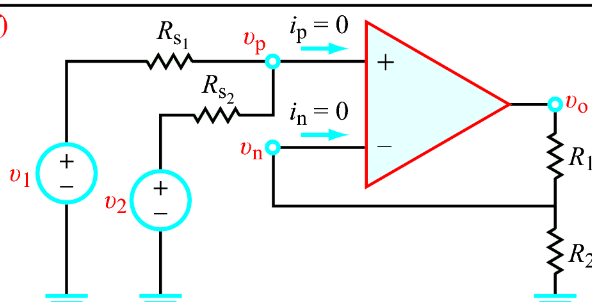
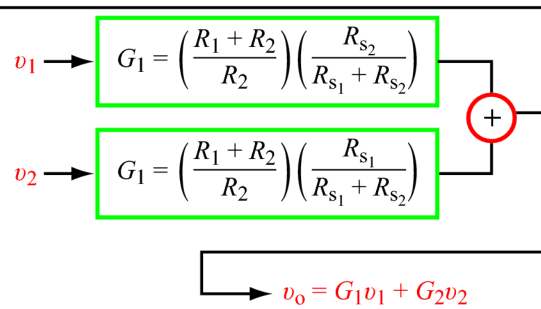
Op-Amp Circuit	Block Diagram
(a) 	 **Noninverting Amp** (v_o independent of R_s)
(b) 	 **Inverting Amp**
(c) 	 **Inverting Summing Amp**
(d) 	 **Subtracting Amp**
(e) 	 **Voltage Follower / Buffer** (v_o independent of R_s and R_L)
(f) 	 **Noninverting Summing Amp**

Table 4-4 **Correspondence between binary sequence and decimal value for a 4-bit digital signal and output of a DAC with $G = -0.5$.**

$V_1V_2V_3V_4$	Decimal Value	DAC Output (V)
0000	0	0
0001	1	-0.5
0010	2	-1
0011	3	-1.5
0100	4	-2
0101	5	-2.5
0110	6	-3
0111	7	-3.5
1000	8	-4
1001	9	-4.5
1010	10	-5
1011	11	-5.5
1100	12	-6
1101	13	-6.5
1110	14	-7
1111	15	-7.5

Table 4-5 List of Multisim components for the circuit in Fig. 4-35.

Component	Group	Family	Quantity	Description
1.5 k	Basic	Resistor	7	1.5 kΩ resistor
15 k	Basic	Resistor	2	15 kΩ resistor
3 k	Basic	Variable resistor	1	3 kΩ resistor
OP_AMP_5T_VIRTUAL	Analog	Analog_Virtual	3	Ideal op amp with 5 terminals
AC_POWER	Sources	Power_Sources	1	1 V ac source, 60 Hz
VDD	Sources	Power_Sources	1	15 V supply
VSS	Sources	Power_Sources	1	−15 V supply

Table 4-6 **Components for the circuit in Fig. 4-37.**

Component	Group	Family	Quantity	Description
MOS_N	Transistors	Transistors_VIRTUAL	1	3-terminal N-MOSFET
MOS_P	Transistors	Transistors_VIRTUAL	1	3-terminal P-MOSFET
VDD	Sources	Power Sources	1	2.5 V supply
GND	Sources	Power Sources	2	Ground node

Table 5-1 **Common nonperiodic waveforms.**

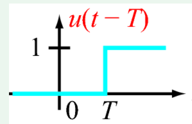
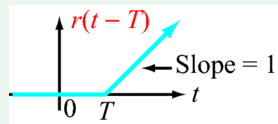
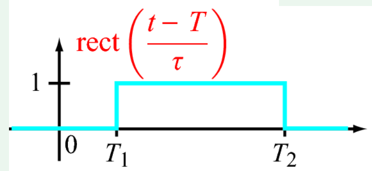
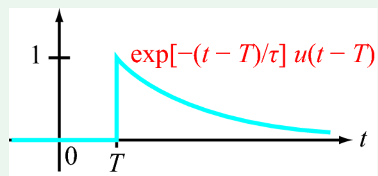
<table>
<tr><th>waveform</th><th>Expression</th><th>General Shape</th></tr>
<tr><td>Step</td><td>$$u(t-T)=\begin{cases}0 & \text{for } t<T\\ 1 & \text{for } t>T\end{cases}$$</td><td>
</td></tr>
<tr><td>Ramp</td><td>$$r(t-T)=(t-T)\,u(t-T)$$</td><td>
</td></tr>
<tr><td>Rectangle</td><td>$$\text{rect}\left(\frac{t-T}{\tau}\right)=u(t-T_1)-u(t-T_2)$$ $$T_1=T-\frac{\tau}{2}; \quad T_2=T+\frac{\tau}{2}$$</td><td>
</td></tr>
<tr><td>Exponential</td><td>$$\exp[-(t-T)/\tau]\,u(t-T)$$</td><td>
</td></tr>
</table>

Table 5-2 **Relative electrical permittivity of common insulators:** $\varepsilon_r = \varepsilon/\varepsilon_0$ **and** $\varepsilon_0 = 8.854 \times 10^{-12}$ **F/m.**

Material	Relative Permittivity ε_r
Air (at sea level)	1.0006
Teflon	2.1
Polystyrene	2.6
Paper	2–4
Glass	4.5–10
Quartz	3.8–5
Bakelite	5
Mica	5.4–6
Porcelain	5.7

Table 5-3 Relative magnetic permeability of materials, $\mu_r = \mu/\mu_0$ and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Material	Relative Permeability μ_r
All Dielectrics and Non-Ferromagnetic Metals	≈ 1.0
Ferromagnetic Metals	
Cobalt	250
Nickel	600
Mild steel	2,000
Iron (pure)	4,000–5,000
Silicon iron	7,000
Mumetal	$\sim 100,000$
Purified iron	$\sim 200,000$

Table 5-4 **Basic properties of R, L, and C.**

Property	R	L	C
i–υ relation	$i = \dfrac{\upsilon}{R}$	$i = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} \upsilon\, dt' + i(t_0)$	$i = C\,\dfrac{d\upsilon}{dt}$
υ-i relation	$\upsilon = iR$	$\upsilon = L\,\dfrac{di}{dt}$	$\upsilon = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i\, dt' + \upsilon(t_0)$
p (power transfer in)	$p = i^2 R$	$p = Li\,\dfrac{di}{dt}$	$p = C\upsilon\,\dfrac{d\upsilon}{dt}$
w (stored energy)	0	$w = \dfrac{1}{2}Li^2$	$w = \dfrac{1}{2}C\upsilon^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$
Parallel combination	$\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$	$\dfrac{1}{L_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can υ change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

Table 5-5 **Response forms of basic first-order circuits.**

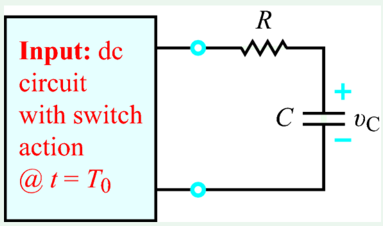
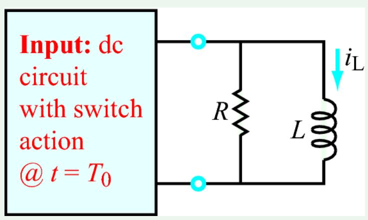
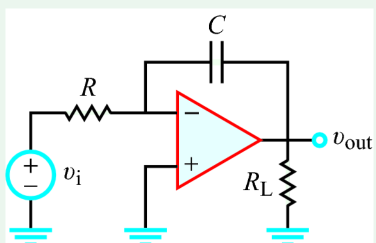
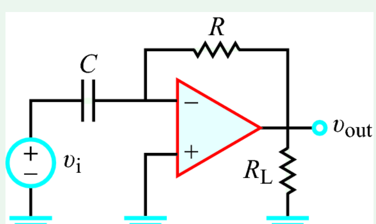
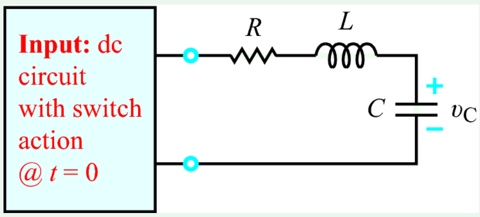
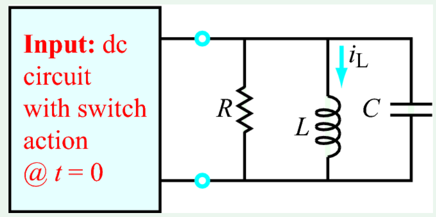
Circuit	Diagram	Response
RC		$\upsilon_C(t) = \left\{\upsilon_C(\infty) + [\upsilon_C(T_0) - \upsilon_C(\infty)]e^{-(t-T_0)/\tau}\right\} u(t - T_0)$ $(\tau = RC)$
RL		$i_L(t) = \left\{i_L(\infty) + [i_L(T_0) - i_L(\infty)]e^{-(t-T_0)/\tau}\right\} u(t - T_0)$ $(\tau = L/R)$
Ideal integrator		$\upsilon_{out}(t) = -\dfrac{1}{RC}\displaystyle\int_{t_0}^{t} \upsilon_i \, dt' + \upsilon_{out}(t_0)$
Ideal differentiator		$\upsilon_{out}(t) = -RC\,\dfrac{d\upsilon_i}{dt}$

Table 5-6 Multisim component list for the circuit in Fig. 5-52.

Component	Group	Family	Quantity	Description
1 k	Basic	Resistor	1	1 kΩ resistor
10 k	Basic	Resistor	1	10 kΩ resistor
5 f	Basic	Capacitor	1	5 fF capacitor
VOLTAGE_CONTROLLED_SPST	Basic	Switch	1	Switch
DC_POWER	Sources	Power_Sources	1	2.5 V dc source
PULSE_VOLTAGE	Sources	Signal_Voltage_Source	1	Pulse-generating voltage source

Table 6-1 **Step response of RLC circuits for $t \geq 0$.**

Series RLC	Parallel RLC
	
Total Response	**Total Response**
Overdamped ($\alpha > \omega_0$) $\upsilon_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + \upsilon_C(\infty)$ $A_1 = \dfrac{\frac{1}{C}\, i_C(0) - s_2[\upsilon_C(0) - \upsilon_C(\infty)]}{s_1 - s_2}$ $A_2 = \left[\dfrac{\frac{1}{C}\, i_C(0) - s_1[\upsilon_C(0) - \upsilon_C(\infty)]}{s_2 - s_1}\right]$	**Overdamped** ($\alpha > \omega_0$) $i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + i_L(\infty)$ $A_1 = \dfrac{\frac{1}{L}\, \upsilon_L(0) - s_2[i_L(0) - i_L(\infty)]}{s_1 - s_2}$ $A_2 = \left[\dfrac{\frac{1}{L}\, \upsilon_L(0) - s_1[i_L(0) - i_L(\infty)]}{s_2 - s_1}\right]$
Critically Damped ($\alpha = \omega_0$) $\upsilon_C(t) = (B_1 + B_2 t)e^{-\alpha t} + \upsilon_C(\infty)$ $B_1 = \upsilon_C(0) - \upsilon_C(\infty)$ $B_2 = \frac{1}{C}\, i_C(0) + \alpha[\upsilon_C(0) - \upsilon_C(\infty)]$	**Critically Damped** ($\alpha = \omega_0$) $i_L(t) = (B_1 + B_2 t)e^{-\alpha t} + i_L(\infty)$ $B_1 = i_L(0) - i_L(\infty)$ $B_2 = \frac{1}{L}\, \upsilon_L(0) + \alpha[i_L(0) - i_L(\infty)]$
Underdamped ($\alpha < \omega_0$) $\upsilon_C(t) = e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + \upsilon_C(\infty)$ $D_1 = \upsilon_C(0) - \upsilon_C(\infty)$ $D_2 = \dfrac{\frac{1}{C}\, i_C(0) + \alpha[\upsilon_C(0) - \upsilon_C(\infty)]}{\omega_d}$	**Underdamped** ($\alpha < \omega_0$) $i_L(t) = e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + i_L(\infty)$ $D_1 = i_L(0) - i_L(\infty)$ $D_2 = \dfrac{\frac{1}{L}\, \upsilon_L(0) + \alpha[i_L(0) - i_L(\infty)]}{\omega_d}$

Auxiliary Relations

$$\alpha = \begin{cases} \dfrac{R}{2L} & \text{Series RLC} \\ \dfrac{1}{2RC} & \text{Parallel RLC} \end{cases} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Table 6-2 **General solution for second-order circuits for $t \geq 0$.**

$x(t)$ = unknown variable (voltage or current)
Differential equation: $x'' + ax' + bx = c$
Initial conditions: $x(0)$ and $x'(0)$
Final condition: $x(\infty) = \dfrac{c}{b}$
$\alpha = \dfrac{a}{2}$ $\quad\omega_0 = \sqrt{b}$
Overdamped Response $\alpha > \omega_0$
$x(t) = [A_1 e^{s_1 t} + A_2 e^{s_2 t} + x(\infty)]\ u(t)$
$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ $\quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$A_1 = \dfrac{x'(0) - s_2[x(0) - x(\infty)]}{s_1 - s_2}$ $\quad A_2 = -\left[\dfrac{x'(0) - s_1[x(0) - x(\infty)]}{s_1 - s_2}\right]$
Critically Damped $\alpha = \omega_0$
$x(t) = [(B_1 + B_2 t)e^{-\alpha t} + x(\infty)]\ u(t)$
$B_1 = x(0) - x(\infty)$ $\quad B_2 = x'(0) + \alpha[x(0) - x(\infty)]$
Underdamped $\alpha < \omega_0$
$x(t) = [D_1 \cos\omega_{\mathrm{d}} t + D_2 \sin\omega_{\mathrm{d}} t + x(\infty)]e^{-\alpha t}\ u(t)$
$D_1 = x(0) - x(\infty)$ $\quad D_2 = \dfrac{x'(0) + \alpha[x(0) - x(\infty)]}{\omega_{\mathrm{d}}}$
$\omega_{\mathrm{d}} = \sqrt{\omega_0^2 - \alpha^2}$

Table 6-3 Component values for the circuit in Fig. 6-19.

Component	Group	Family	Quantity	Description
1	Basic	Resistor	1	1 Ω resistor
300 m	Basic	Inductor	1	300 mH inductor
5.33 m	Basic	Capacitor	1	5.33 mF capacitor
SPDT	Basic	Switch	1	Single-pole double-throw (SPDT) switch
DC_POWER	Sources	Power_Sources	1	1 V dc source

Table 6-4 **Parts for the Multisim circuit in Fig. 6-23.**

Component	Group	Family	Quantity	Description
TS_IDEAL	Basic	Transformer	1	1 mH:1 mH ideal transformer
1 k	Basic	Resistor	1	1 kΩ resistor
1 μ	Basic	Capacitor	1	1 μF capacitor
SPDT	Basic	Switch	1	SPDT switch
AC_CURRENT	Sources	Signal_Current_Source	1	1 mA, 5.033 kHz

Table 7-1 Useful trigonometric identities (additional relations are given in Appendix D).

$\sin x = \pm\cos(x \mp 90^\circ)$	(7.7a)
$\cos x = \pm\sin(x \pm 90^\circ)$	(7.7b)
$\sin x = -\sin(x \pm 180^\circ)$	(7.7c)
$\cos x = -\cos(x \pm 180^\circ)$	(7.7d)
$\sin(-x) = -\sin x$	(7.7e)
$\cos(-x) = \cos x$	(7.7f)
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	(7.7g)
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	(7.7h)
$2\sin x \sin y = \cos(x-y) - \cos(x+y)$	(7.7i)
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	(7.7j)
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	(7.7k)

Table 7-2 Properties of complex numbers.

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$

$\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$	$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
$\mathbf{z} = x + jy = \lvert\mathbf{z}\rvert e^{j\theta}$	$\mathbf{z}^* = x - jy = \lvert\mathbf{z}\rvert e^{-j\theta}$
$x = \mathfrak{Re}(\mathbf{z}) = \lvert\mathbf{z}\rvert\cos\theta$	$\lvert\mathbf{z}\rvert = \sqrt{\mathbf{z}\mathbf{z}^*} = \sqrt{x^2+y^2}$
$y = \mathfrak{Im}(\mathbf{z}) = \lvert\mathbf{z}\rvert\sin\theta$	$\theta = \begin{cases} \tan^{-1}(y/x) & \text{if } x > 0, \\ \tan^{-1}(y/x) \pm \pi & \text{if } x < 0, \\ \pi/2 & \text{if } x = 0 \text{ and } y > 0, \\ -\pi/2 & \text{if } x = 0 \text{ and } y < 0. \end{cases}$
$\mathbf{z}^n = \lvert\mathbf{z}\rvert^n e^{jn\theta}$	$\mathbf{z}^{1/2} = \pm\lvert\mathbf{z}\rvert^{1/2} e^{j\theta/2}$
$\mathbf{z}_1 = x_1 + jy_1$	$\mathbf{z}_2 = x_2 + jy_2$
$\mathbf{z}_1 = \mathbf{z}_2$ iff $x_1 = x_2$ and $y_1 = y_2$	$\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2)$
$\mathbf{z}_1\mathbf{z}_2 = \lvert\mathbf{z}_1\rvert\lvert\mathbf{z}_2\rvert e^{j(\theta_1+\theta_2)}$	$\dfrac{\mathbf{z}_1}{\mathbf{z}_2} = \dfrac{\lvert\mathbf{z}_1\rvert}{\lvert\mathbf{z}_2\rvert} e^{j(\theta_1-\theta_2)}$
$-1 = e^{j\pi} = e^{-j\pi} = 1\angle\pm 180^\circ$	
$j = e^{j\pi/2} = 1\angle 90^\circ$	$-j = e^{-j\pi/2} = 1\angle -90^\circ$
$\sqrt{j} = \pm e^{j\pi/4} = \pm\dfrac{(1+j)}{\sqrt{2}}$	$\sqrt{-j} = \pm e^{-j\pi/4} = \pm\dfrac{(1-j)}{\sqrt{2}}$

Table 7-3 **Time-domain sinusoidal functions $x(t)$ and their cosine-reference phasor-domain counterparts $\mathbf{X}$, where $x(t) = \mathfrak{Re}[\mathbf{X}e^{j\omega t}]$.**

$x(t)$		$\mathbf{X}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j\phi}$
$-A\cos(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi\pm\pi)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2} = -jA$
$A\sin(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi-\pi/2)}$
$-A\sin(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi+\pi/2)}$
$\frac{d}{dt}(x(t))$	$\leftrightarrow$	$j\omega\mathbf{X}$
$\frac{d}{dt}[A\cos(\omega t+\phi)]$	$\leftrightarrow$	$j\omega Ae^{j\phi}$
$\int x(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\,\mathbf{X}$
$\int A\cos(\omega t+\phi)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\,Ae^{j\phi}$

Table 7-4 **Summary of υ–i properties for R, L, and C.**

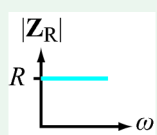
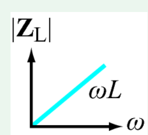
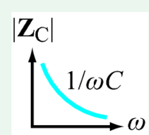
Property	R	L	C
υ–i	$\upsilon = Ri$	$\upsilon = L\,\dfrac{di}{dt}$	$i = C\,\dfrac{d\upsilon}{dt}$
V–I	$\mathbf{V} = R\mathbf{I}$	$\mathbf{V} = j\omega L\mathbf{I}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$
Z	R	$j\omega L$	$\dfrac{1}{j\omega C}$
dc equivalent	R	Short circuit	Open circuit
High-frequency equivalent	R	Open circuit	Short circuit
Frequency response			

Table 7-5 **Inverting amplifier gain G as a function of oscillation frequency f. $G_{\text{ideal}} = -5$.**

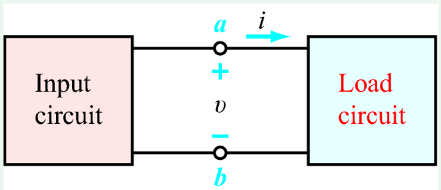
f (Hz)	A	G	Error
0 (dc)	10^5	-4.997	0.06%
100	10^4	-4.970	0.6%
1 k	10^3	-4.714	5.7%
10 k	10^2	-3.111	37.8%
100 k	10	-0.707	85.9%
1 M	1	-0.081	98.4%

The error is defined as

$$\% \text{ error} = \left(\frac{G_{\text{ideal}} - G}{G_{\text{ideal}}} \right) \times 100.$$

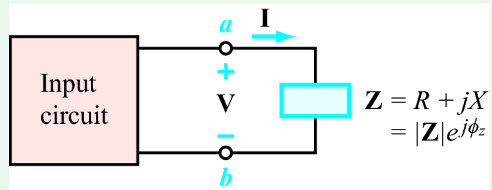
Table 8-1 **Summary of power-related quantities.**

Time Domain



$$\upsilon(t) = V_{\mathrm{m}} \cos(\omega t + \phi_\upsilon)$$
$$i(t) = I_{\mathrm{m}} \cos(\omega t + \phi_i)$$
$$V_{\mathrm{rms}} = V_{\mathrm{m}}/\sqrt{2}$$
$$I_{\mathrm{rms}} = I_{\mathrm{m}}/\sqrt{2}$$

Phasor Domain



$$\mathbf{V} = V_{\mathrm{m}} e^{j\phi_\upsilon}$$
$$\mathbf{I} = I_{\mathrm{m}} e^{j\phi_i}$$
$$\mathbf{V}_{\mathrm{rms}} = V_{\mathrm{rms}} e^{j\phi_\upsilon}$$
$$\mathbf{I}_{\mathrm{rms}} = I_{\mathrm{rms}} e^{j\phi_i}$$

Complex Power

$$\mathbf{S} = \tfrac{1}{2}\,\mathbf{V}\mathbf{I}^* = \mathbf{V}_{\mathrm{rms}}\mathbf{I}^*_{\mathrm{rms}} = P_{\mathrm{av}} + jQ$$

Real Average Power

$$\begin{aligned} P_{\mathrm{av}} &= \mathfrak{Re}[\mathbf{S}] \\ &= V_{\mathrm{rms}} I_{\mathrm{rms}} \cos(\phi_\upsilon - \phi_i) \\ &= I^2_{\mathrm{rms}} R = V^2_{\mathrm{rms}} R/|\mathbf{Z}|^2 \end{aligned}$$

Reactive Power

$$\begin{aligned} Q &= \mathfrak{Im}[\mathbf{S}] \\ &= V_{\mathrm{rms}} I_{\mathrm{rms}} \sin(\phi_\upsilon - \phi_i) \\ &= I^2_{\mathrm{rms}} X = V^2_{\mathrm{rms}} X/|\mathbf{Z}|^2 \end{aligned}$$

Apparent Power

$$\begin{aligned} S = |\mathbf{S}| &= \sqrt{P^2_{\mathrm{av}} + Q^2} \\ &= V_{\mathrm{rms}} I_{\mathrm{rms}} \\ &= I^2_{\mathrm{rms}} |\mathbf{Z}| = V^2_{\mathrm{rms}}/|\mathbf{Z}| \end{aligned}$$

$$\mathbf{S} = S e^{j(\phi_\upsilon - \phi_i)} = S e^{j\phi_z}$$
$$\phi_z = \phi_\upsilon - \phi_i$$

Power Factor

$$\begin{aligned} pf &= \frac{P_{\mathrm{av}}}{S} \\ &= \cos(\phi_\upsilon - \phi_i) \\ &= \cos\phi_z \end{aligned}$$

Table 8-2 **Power factor leading and lagging relationships for a load $\mathbf{Z} = R + jX$.**

Load Type	$\phi_z = \phi_v - \phi_i$	I-V Relationship	*pf*
Purely Resistive ($X = 0$)	$\phi_z = 0$	**I** in phase with **V**	1
Inductive ($X > 0$)	$0 < \phi_z \leq 90^\circ$	**I** lags **V**	lagging
Purely Inductive ($X > 0$ and $R = 0$)	$\phi_z = 90^\circ$	**I** lags **V** by 90°	lagging
Capacitive ($X < 0$)	$-90^\circ \leq \phi_z < 0$	**I** leads **V**	leading
Purely Capacitive ($X < 0$ and $R = 0$)	$\phi_z = -90^\circ$	**I** leads **V** by 90°	leading

Table 9-1 Correspondence between power ratios in natural numbers and their dB values (left table) and between voltage or current ratios and their dB values (right table).

$\dfrac{P}{P_0}$	dB
10^N	$10N$ dB
10^3	30 dB
100	20 dB
10	10 dB
4	≈ 6 dB
2	≈ 3 dB
1	0 dB
0.5	≈ -3 dB
0.25	≈ -6 dB
0.1	-10 dB
10^{-N}	$-10N$ dB

$\left\lvert\dfrac{\mathbf{V}}{\mathbf{V}_0}\right\rvert$ or $\left\lvert\dfrac{\mathbf{I}}{\mathbf{I}_0}\right\rvert$	dB
10^N	$20N$ dB
10^3	60 dB
100	40 dB
10	20 dB
4	≈ 12 dB
2	≈ 6 dB
1	0 dB
0.5	≈ -6 dB
0.25	≈ -12 dB
0.1	-20 dB
10^{-N}	$-20N$ dB

Table 9-2 **Bode straight-line approximations for magnitude and phase.**

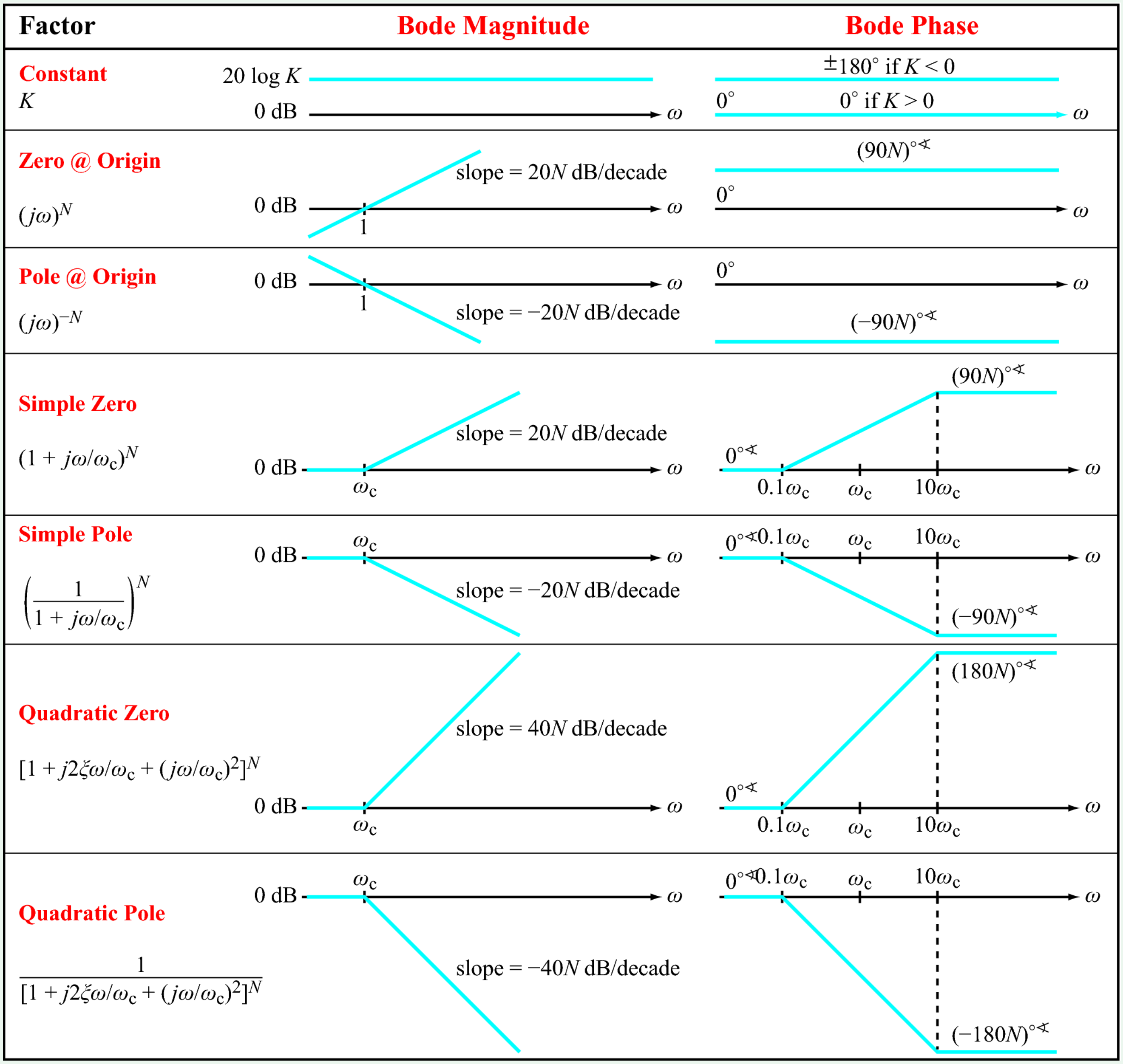
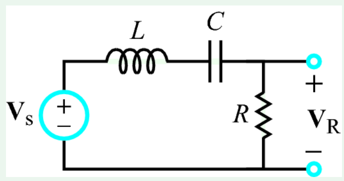
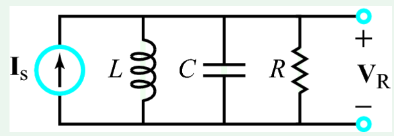


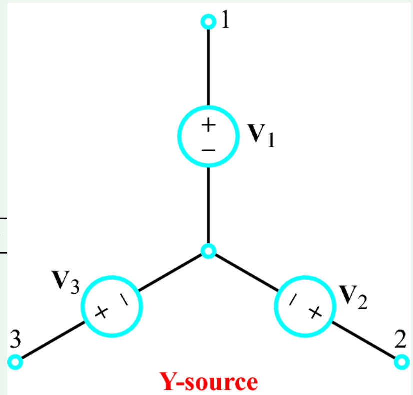
Table 9-3 Attributes of series and parallel RLC bandpass circuits.

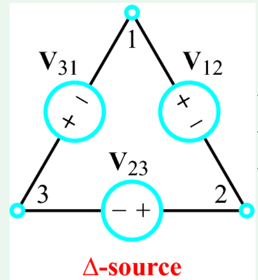
	Series	Parallel
RLC Circuit		
Transfer Function	$\mathbf{H} = \dfrac{\mathbf{V}_R}{\mathbf{V}_s}$	$\mathbf{H} = \dfrac{\mathbf{V}_R}{\mathbf{I}_s}$
Resonant Frequency, ω_0	$\dfrac{1}{\sqrt{LC}}$	$\dfrac{1}{\sqrt{LC}}$
Bandwidth, B	$\dfrac{R}{L}$	$\dfrac{1}{RC}$
Quality Factor, Q	$\dfrac{\omega_0}{B} = \dfrac{\omega_0 L}{R}$	$\dfrac{\omega_0}{B} = \dfrac{R}{\omega_0 L}$
Lower Half-Power Frequency, ω_{c_1}	$\left[-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$	$\left[-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$
Upper Half-Power Frequency, ω_{c_2}	$\left[\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$	$\left[\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$

Notes: (1) The expression for Q of the series RLC circuit is the inverse of that for Q of the parallel circuit.
(2) For $Q \geq 10$, $\omega_{c_1} \approx \omega_0 - \frac{B}{2}$, and $\omega_{c_2} \approx \omega_0 + \frac{B}{2}$.

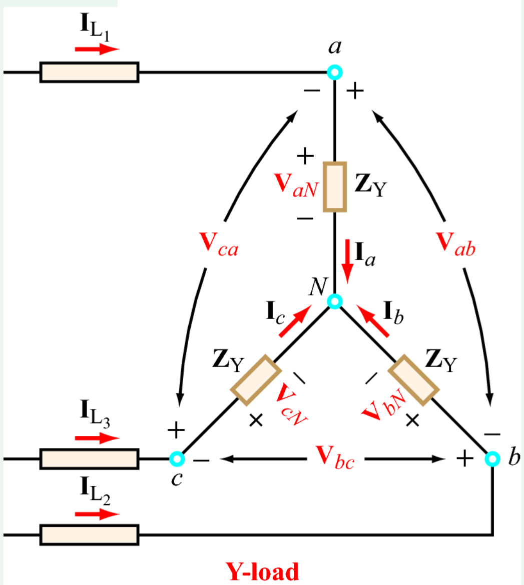
Table 10-1 Balanced networks.

$\mathbf{V}_1 = V_{Ys}\,\underline{/\phi_1}$
$\mathbf{V}_2 = V_{Ys}\,\underline{/\phi_1 -}$
$\mathbf{V}_3 = V_{Ys}\,\underline{/\phi_1 +}$

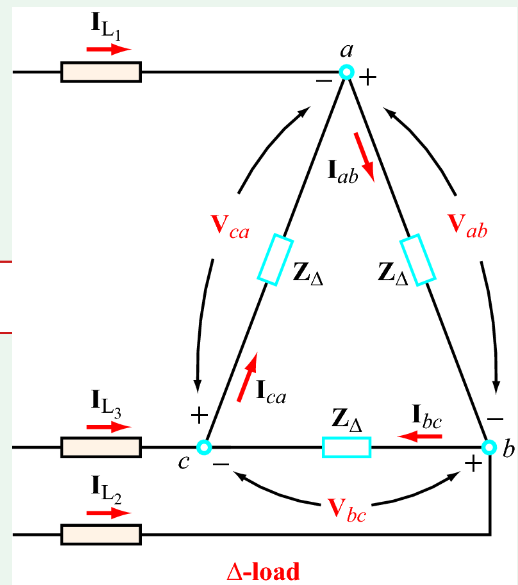




$\mathbf{V}_{12} = \mathbf{V}_1 \times \sqrt{3}\,\underline{/30^\circ}$
$\mathbf{V}_{23} = \mathbf{V}_2 \times \sqrt{3}\,\underline{/30^\circ}$
$\mathbf{V}_{31} = \mathbf{V}_3 \times \sqrt{3}\,\underline{/30^\circ}$



$\mathbf{Z}_\Delta = 3\mathbf{Z}_\mathrm{Y}$



$\mathbf{V}_{aN} = (V_\mathrm{L}/\sqrt{3})\,\underline{/0^\circ}$

$\mathbf{V}_{bN} = (V_\mathrm{L}/\sqrt{3})\,\underline{/-120^\circ}$

$\mathbf{V}_{cN} = (V_\mathrm{L}/\sqrt{3})\,\underline{/-240^\circ}$

V_L = rms magnitude of line-to-line voltage

$I_\mathrm{L} = \sqrt{3}\,V_\mathrm{L}/|\mathbf{Z}_\mathrm{Y}|$

$\mathbf{S}_\mathrm{T} = \sqrt{3}\,V_\mathrm{L} I_\mathrm{L}\,\underline{/\phi_\mathrm{Y}}$

$\mathbf{V}_{ab} = V_\mathrm{L}\,\underline{/30^\circ}^{\dagger}$

$\mathbf{V}_{bc} = V_\mathrm{L}\,\underline{/-90^\circ}$

$\mathbf{V}_{ca} = V_\mathrm{L}\,\underline{/150^\circ}$

$I_\mathrm{L} = \sqrt{3}\,V_\mathrm{L}/|\mathbf{Z}_\Delta|$

$\mathbf{S}_\mathrm{T} = \sqrt{3}\,V_\mathrm{L} I_\mathrm{L}\,\underline{/\phi_\Delta}$

† Phase relative to $\mathbf{V}_{aN}$ of Y-load

Table 12-1 **Properties of the Laplace transform ($f(t) = 0$ for $t < 0^-$).**

Property	$f(t)$		$\mathbf{F}(\mathbf{s}) = \mathscr{L}[f(t)]$
1. Multiplication by constant	$K\, f(t)$	$\leftrightarrow$	$K\, \mathbf{F}(\mathbf{s})$
2. Linearity	$K_1\, f_1(t) + K_2\, f_2(t)$	$\leftrightarrow$	$K_1\, \mathbf{F}_1(\mathbf{s}) + K_2\, \mathbf{F}_2(\mathbf{s})$
3. Time scaling	$f(at), \quad a > 0$	$\leftrightarrow$	$\dfrac{1}{a}\, \mathbf{F}\left(\dfrac{\mathbf{s}}{a}\right)$
4. Time shift	$f(t-T)\, u(t-T)$	$\leftrightarrow$	$e^{-T\mathbf{s}}\, \mathbf{F}(\mathbf{s}), \quad T \geq 0$
5. Frequency shift	$e^{-at}\, f(t)$	$\leftrightarrow$	$\mathbf{F}(\mathbf{s}+a)$
6. Time 1st derivative	$f' = \dfrac{df}{dt}$	$\leftrightarrow$	$\mathbf{s}\, \mathbf{F}(\mathbf{s}) - f(0^-)$
7. Time 2nd derivative	$f'' = \dfrac{d^2 f}{dt^2}$	$\leftrightarrow$	$\mathbf{s}^2 \mathbf{F}(\mathbf{s}) - \mathbf{s} f(0^-) - f'(0^-)$
8. Time integral	$\displaystyle\int_{0^-}^{t} f(\tau)\, d\tau$	$\leftrightarrow$	$\dfrac{1}{\mathbf{s}}\, \mathbf{F}(\mathbf{s})$
9. Frequency derivative	$t\, f(t)$	$\leftrightarrow$	$-\dfrac{d}{d\mathbf{s}}\, \mathbf{F}(\mathbf{s})$
10. Frequency integral	$\dfrac{f(t)}{t}$	$\leftrightarrow$	$\displaystyle\int_{\mathbf{s}}^{\infty} \mathbf{F}(\mathbf{s}')\, d\mathbf{s}'$

Table 12-2 **Examples of Laplace transform pairs for $T \geq 0$. Note that multiplication by $u(t)$ guarantees that $f(t) = 0$ for $t < 0^-$.**

	Laplace Transform Pairs		
	$f(t)$		$\mathbf{F}(\mathbf{s}) = \mathscr{L}[f(t)]$
1	$\delta(t)$	$\leftrightarrow$	1
1a	$\delta(t-T)$	$\leftrightarrow$	$e^{-T\mathbf{s}}$
2	1 or $u(t)$	$\leftrightarrow$	$\dfrac{1}{\mathbf{s}}$
2a	$u(t-T)$	$\leftrightarrow$	$\dfrac{e^{-T\mathbf{s}}}{\mathbf{s}}$
3	$e^{-at}\, u(t)$	$\leftrightarrow$	$\dfrac{1}{\mathbf{s}+a}$
3a	$e^{-a(t-T)}\, u(t-T)$	$\leftrightarrow$	$\dfrac{e^{-T\mathbf{s}}}{\mathbf{s}+a}$
4	$t\, u(t)$	$\leftrightarrow$	$\dfrac{1}{\mathbf{s}^2}$
4a	$(t-T)\, u(t-T)$	$\leftrightarrow$	$\dfrac{e^{-T\mathbf{s}}}{\mathbf{s}^2}$
5	$t^2\, u(t)$	$\leftrightarrow$	$\dfrac{2}{\mathbf{s}^3}$
6	$te^{-at}\, u(t)$	$\leftrightarrow$	$\dfrac{1}{(\mathbf{s}+a)^2}$
7	$t^2e^{-at}\, u(t)$	$\leftrightarrow$	$\dfrac{2}{(\mathbf{s}+a)^3}$
8	$t^{n-1}e^{-at}\, u(t)$	$\leftrightarrow$	$\dfrac{(n-1)!}{(\mathbf{s}+a)^n}$
9	$\sin \omega t\, u(t)$	$\leftrightarrow$	$\dfrac{\omega}{\mathbf{s}^2+\omega^2}$
10	$\sin(\omega t+\theta)\, u(t)$	$\leftrightarrow$	$\dfrac{\mathbf{s}\sin\theta+\omega\cos\theta}{\mathbf{s}^2+\omega^2}$
11	$\cos \omega t\, u(t)$	$\leftrightarrow$	$\dfrac{\mathbf{s}}{\mathbf{s}^2+\omega^2}$
12	$\cos(\omega t+\theta)\, u(t)$	$\leftrightarrow$	$\dfrac{\mathbf{s}\cos\theta-\omega\sin\theta}{\mathbf{s}^2+\omega^2}$
13	$e^{-at}\sin \omega t\, u(t)$	$\leftrightarrow$	$\dfrac{\omega}{(\mathbf{s}+a)^2+\omega^2}$
14	$e^{-at}\cos \omega t\, u(t)$	$\leftrightarrow$	$\dfrac{\mathbf{s}+a}{(\mathbf{s}+a)^2+\omega^2}$
15	$2e^{-at}\cos(bt-\theta)\, u(t)$	$\leftrightarrow$	$\dfrac{e^{j\theta}}{\mathbf{s}+a+jb}+\dfrac{e^{-j\theta}}{\mathbf{s}+a-jb}$
16	$\dfrac{2t^{n-1}}{(n-1)!}\, e^{-at}\cos(bt-\theta)\, u(t)$	$\leftrightarrow$	$\dfrac{e^{j\theta}}{(\mathbf{s}+a+jb)^n}+\dfrac{e^{-j\theta}}{(\mathbf{s}+a-jb)^n}$

Note: $(n-1)! = (n-1)(n-2)\ldots 1$.

Table 12-3 **Transform pairs for four types of poles.**

Pole	$\mathbf{F}(\mathbf{s})$	$f(t)$
1. Distinct real	$\dfrac{A}{\mathbf{s}+a}$	$Ae^{-at}\,u(t)$
2. Repeated real	$\dfrac{A}{(\mathbf{s}+a)^n}$	$A\,\dfrac{t^{n-1}}{(n-1)!}\,e^{-at}\,u(t)$
3. Distinct complex	$\left[\dfrac{Ae^{j\theta}}{\mathbf{s}+a+jb}+\dfrac{Ae^{-j\theta}}{\mathbf{s}+a-jb}\right]$	$2Ae^{-at}\cos(bt-\theta)\,u(t)$
4. Repeated complex	$\left[\dfrac{Ae^{j\theta}}{(\mathbf{s}+a+jb)^n}+\dfrac{Ae^{-j\theta}}{(\mathbf{s}+a-jb)^n}\right]$	$\dfrac{2At^{n-1}}{(n-1)!}\,e^{-at}\cos(bt-\theta)\,u(t)$

Table 12-4 **Circuit models for R, L, and C in the s-domain.**

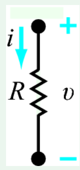
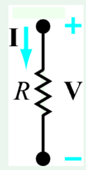
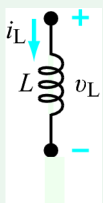
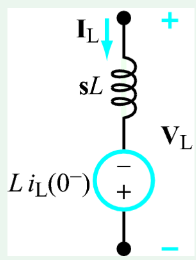
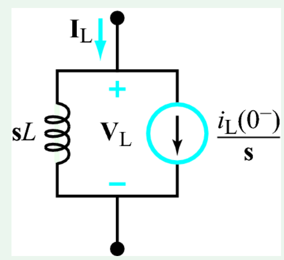
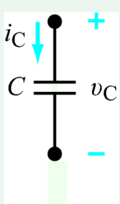
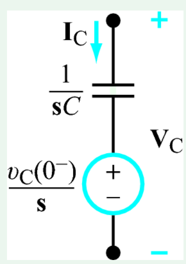
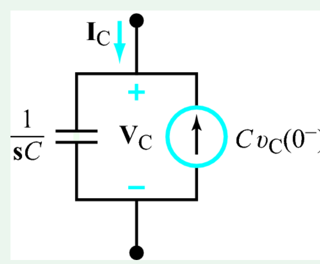
Time-Domain	s-Domain		
Resistor  $\upsilon = Ri$	 $\mathbf{V} = R\mathbf{I}$		
Inductor  $\upsilon_L = L\,\dfrac{di_L}{dt}$ $i_L = \dfrac{1}{L}\displaystyle\int_{0^-}^{t} \upsilon_L\,dt + i_L(0^-)$	 $\mathbf{V}_L = \mathbf{s}L\mathbf{I}_L - L\,i_L(0^-)$	**OR**	 $\mathbf{I}_L = \dfrac{\mathbf{V}_L}{\mathbf{s}L} + \dfrac{i_L(0^-)}{\mathbf{s}}$
Capacitor  $i_C = C\,\dfrac{d\upsilon_C}{dt}$ $\upsilon_C = \dfrac{1}{C}\displaystyle\int_{0^-}^{t} i_C\,dt + \upsilon_C(0^-)$	 $\mathbf{V}_C = \dfrac{\mathbf{I}_C}{\mathbf{s}C} + \dfrac{\upsilon_C(0^-)}{\mathbf{s}}$	**OR**	 $\mathbf{I}_C = \mathbf{s}C\mathbf{V}_C - C\,\upsilon_C(0^-)$

Table 13-1 **Trigonometric integral properties for any integers m and n. The integration period $T = 2\pi/\omega_0$.**

Property	Integral
1	$\int_0^T \sin n\omega_0 t \, dt = 0$
2	$\int_0^T \cos n\omega_0 t \, dt = 0$
3	$\int_0^T \sin n\omega_0 t \sin m\omega_0 t \, dt = 0, \qquad n \neq m$
4	$\int_0^T \cos n\omega_0 t \cos m\omega_0 t \, dt = 0, \qquad n \neq m$
5	$\int_0^T \sin n\omega_0 t \cos m\omega_0 t \, dt = 0$
6	$\int_0^T \sin^2 n\omega_0 t \, dt = T/2$
7	$\int_0^T \cos^2 n\omega_0 t \, dt = T/2$

Note: All integral properties remain valid when the arguments $n\omega_0 t$ and $m\omega_0 t$ are phase shifted by a constant angle ϕ_0. Thus, Property 1, for example, becomes $\int_0^T \sin(n\omega_0 t + \phi_0) \, dt = 0$, and Property 5 becomes $\int_0^T \sin(n\omega_0 t + \phi_0) \cos(m\omega_0 t + \phi_0) \, dt = 0$.

Table 13-2 Fourier series expressions for a select set of periodic waveforms.

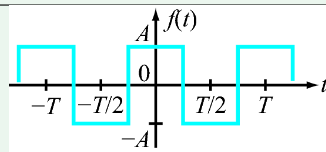
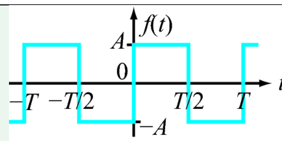
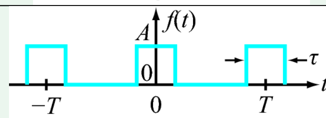
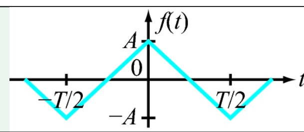
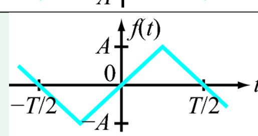
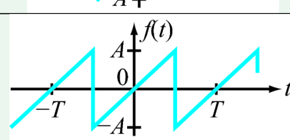
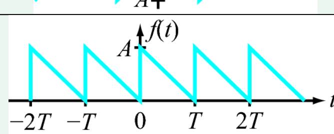
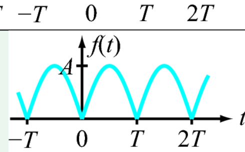
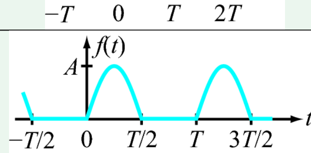
	Waveform	Fourier Series
1. Square Wave		$f(t) = \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos\left(\frac{2n\pi t}{T}\right)$
2. Time-Shifted Square Wave		$f(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4A}{n\pi} \sin\left(\frac{2n\pi t}{T}\right)$
3. Pulse Train		$f(t) = \frac{A\tau}{T} + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T}\right) \cos\left(\frac{2n\pi t}{T}\right)$
4. Triangular Wave		$f(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{8A}{n^2\pi^2} \cos\left(\frac{2n\pi t}{T}\right)$
5. Shifted Triangular Wave		$f(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{8A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{2n\pi t}{T}\right)$
6. Sawtooth		$f(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2A}{n\pi} \sin\left(\frac{2n\pi t}{T}\right)$
7. Backward Sawtooth		$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{A}{n\pi} \sin\left(\frac{2n\pi t}{T}\right)$
8. Full-Wave Rectified Sinusoid		$f(t) = \frac{2A}{\pi} + \sum_{n=1}^{\infty} \frac{4A}{\pi(1-4n^2)} \cos\left(\frac{2n\pi t}{T}\right)$
9. Half-Wave Rectified Sinusoid		$f(t) = \frac{A}{\pi} + \frac{A}{2} \sin\left(\frac{2\pi t}{T}\right) + \sum_{\substack{n=2 \\ n=\text{even}}}^{\infty} \frac{2A}{\pi(1-n^2)} \cos\left(\frac{2n\pi t}{T}\right)$

Table 13-3 **Fourier series representations for a periodic function $f(t)$.**

Cosine/Sine	Amplitude/Phase	Complex Exponential
$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$	$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n)$	$f(t) = \sum_{n=-\infty}^{\infty} \mathbf{c}_n e^{jn\omega_0 t}$
$a_0 = \frac{1}{T} \int_0^T f(t)\ dt$	$A_n e^{j\phi_n} = a_n - jb_n$	$\mathbf{c}_n = \lvert\mathbf{c}_n\rvert e^{j\phi_n};\ \mathbf{c}_{-n} = \mathbf{c}_n^*$
$a_n = \frac{2}{T} \int_0^T f(t)\ \cos n\omega_0 t\ dt$	$A_n = \sqrt{a_n^2 + b_n^2}$	$\lvert\mathbf{c}_n\rvert = A_n/2;\ c_0 = a_0$
$b_n = \frac{2}{T} \int_0^T f(t)\ \sin n\omega_0 t\ dt$	$\phi_n = \begin{cases} -\tan^{-1}\left(\frac{b_n}{a_n}\right), & a_n > 0 \\ \pi - \tan^{-1}\left(\frac{b_n}{a_n}\right), & a_n < 0 \end{cases}$	$\mathbf{c}_n = \frac{1}{T} \int_0^T f(t)\ e^{-jn\omega_0 t}\ dt$
$a_0 = c_0;\ a_n = A_n \cos\phi_n;\ b_n = -A_n \sin\phi_n;\ \mathbf{c}_n = \frac{1}{2}(a_n - jb_n)$		

Table 13-4 Examples of Fourier transform pairs. Note that constant $a \geq 0$.

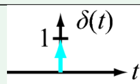
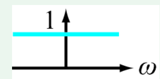
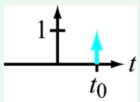
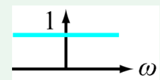
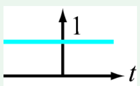
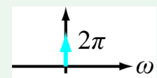
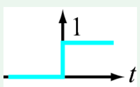
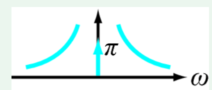
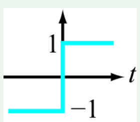
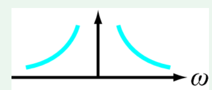
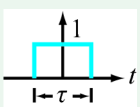
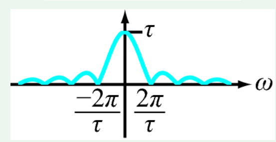
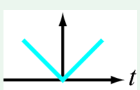
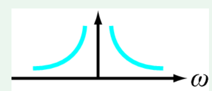
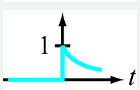
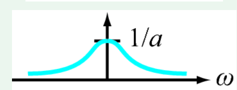
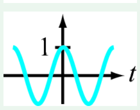
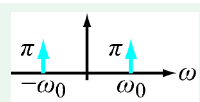
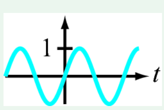
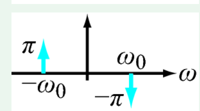
	$f(t)$		$\mathbf{F}(\omega)=\mathscr{F}[f(t)]$		$\lvert\mathbf{F}(\omega)\rvert$
			BASIC FUNCTIONS		
1.		$\delta(t)$	$\leftrightarrow$	1	
1a.		$\delta(t-t_0)$	$\leftrightarrow$	$e^{-j\omega t_0}$	
2.		1	$\leftrightarrow$	$2\pi\,\delta(\omega)$	
3.		$u(t)$	$\leftrightarrow$	$\pi\,\delta(\omega)+1/j\omega$	
4.		$\text{sgn}(t)$	$\leftrightarrow$	$2/j\omega$	
5.		$\text{rect}(t/\tau)$	$\leftrightarrow$	$\tau\,\text{sinc}(\omega\tau/2)$	
6.		$\lvert t\rvert$	$\leftrightarrow$	$-2/\omega^2$	
7.		$e^{-at}\,u(t)$	$\leftrightarrow$	$1/(a+j\omega)$	
8.		$\cos\omega_0 t$	$\leftrightarrow$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
9.		$\sin\omega_0 t$	$\leftrightarrow$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
			ADDITIONAL FUNCTIONS		
10.		$e^{j\omega_0 t}$	$\leftrightarrow$	$2\pi\,\delta(\omega-\omega_0)$	
11.		$te^{-at}\,u(t)$	$\leftrightarrow$	$1/(a+j\omega)^2$	
12.		$[e^{-at}\sin\omega_0 t]\,u(t)$	$\leftrightarrow$	$\omega_0/[(a+j\omega)^2+\omega_0^2]$	
13.		$[e^{-at}\cos\omega_0 t]\,u(t)$	$\leftrightarrow$	$(a+j\omega)/[(a+j\omega)^2+\omega_0^2]$	

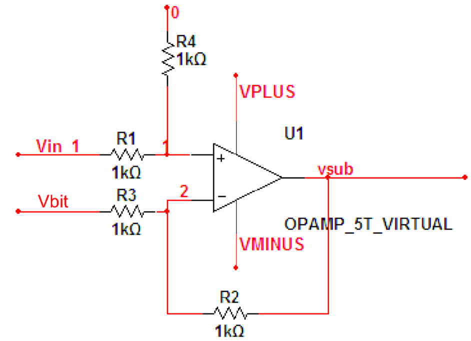
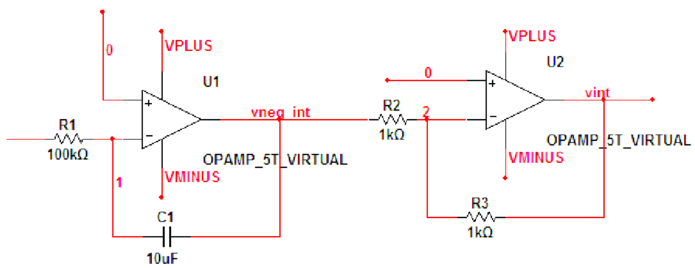
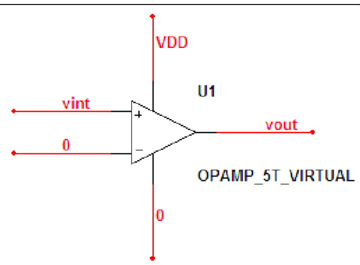
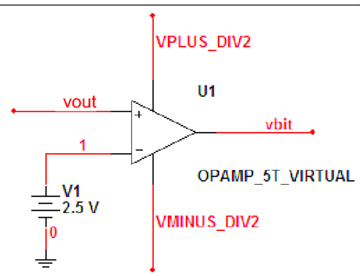
Table 13-5 **Major properties of the Fourier transform.**

Property	$f(t)$		$\mathbf{F}(\omega) = \mathscr{F}[f(t)]$
1. Multiplication by a constant	$K\, f(t)$	$\leftrightarrow$	$K\,\mathbf{F}(\omega)$
2. Linearity	$K_1\, f_1(t) + K_2\, f_2(t)$	$\leftrightarrow$	$K_1\,\mathbf{F}_1(\omega) + K_2\,\mathbf{F}_2(\omega)$
3. Time scaling	$f(at)$	$\leftrightarrow$	$\dfrac{1}{\|a\|}\,\mathbf{F}\left(\dfrac{\omega}{a}\right)$
4. Time shift	$f(t - t_0)$	$\leftrightarrow$	$e^{-j\omega t_0}\,\mathbf{F}(\omega)$
5. Frequency shift	$e^{j\omega_0 t}\, f(t)$	$\leftrightarrow$	$\mathbf{F}(\omega - \omega_0)$
6. Time 1st derivative	$f' = \dfrac{df}{dt}$	$\leftrightarrow$	$j\omega\,\mathbf{F}(\omega)$
7. Time nth derivative	$\dfrac{d^n f}{dt^n}$	$\leftrightarrow$	$(j\omega)^n\,\mathbf{F}(\omega)$
8. Time integral	$\int_{-\infty}^{t} f(t)\, dt$	$\leftrightarrow$	$\dfrac{\mathbf{F}(\omega)}{j\omega} + \pi\,\mathbf{F}(0)\,\delta(\omega)$
9. Frequency derivative	$t^n\, f(t)$	$\leftrightarrow$	$(j)^n \dfrac{d^n\mathbf{F}(\omega)}{d\omega^n}$
10. Modulation	$\cos\omega_0 t\; f(t)$	$\leftrightarrow$	$\frac{1}{2}[\mathbf{F}(\omega - \omega_0) + \mathbf{F}(\omega + \omega_0)]$
11. Convolution in t	$f_1(t) * f_2(t)$	$\leftrightarrow$	$\mathbf{F}_1(\omega)\,\mathbf{F}_2(\omega)$
12. Convolution in ω	$f_1(t)\, f_2(t)$	$\leftrightarrow$	$\dfrac{1}{2\pi}\,\mathbf{F}_1(\omega) * \mathbf{F}_2(\omega)$

Table 13-6 Methods of solution.

Input $x(t)$			
Duration	**Waveform**	**Solution Method**	**Output $y(t)$**
Everlasting	**Sinusoid**	Phasor Domain	Steady-State Component (no transient exists)
Everlasting	**Periodic**	Phasor Domain and Fourier Series	Steady-State Component (no transient exists)
One-sided, $x(t) = 0$, for $t < 0^-$	**Any**	Laplace Transform (unilateral) (can accommodate nonzero initial conditions)	Complete Solution (transient + steady-state)
Everlasting	**Any**	Bilateral Laplace Transform or Fourier Transform	Complete Solution (transient + steady-state)

Table 13-7 **Multisim circuits of the ΣΔ modulator.**

Multisim Circuit	Description and Notes
	**Subtractor:** This is a difference amplifier (following **Table 4-3**) with a voltage gain of 1. VPLUS and VMINUS are the extremes of the analog input (in the complete circuit, they are set to ±12 V).
	**Integrator:** This circuit consists of an inverting integrator amplifier (Section 5-6.1) and an inverting amplifier (following **Table 4-3**) with a voltage gain of 1 (to remove the integrator's negative sign).
	**Comparator:** The comparator is a simple op amp with no feedback (open loop). Since the internal voltage gain A of the op amp is so high (Section 4-1.2), any positive difference between the noninverting and the inverting inputs immediately drives the amplifier output to V_{DD}; a negative difference drives the amplifier output to 0 V. V_{DD} is set to the desired digital voltage level (5 V, in the case of the complete circuit in **Fig. 13-22**).
	**1-Bit Digital-to-Analog Converter (DAC):** The DAC is very similar to the comparator. The input voltage is compared to a voltage level halfway between 0 and V_{DD}; this has the effect of transforming an input of V_{DD} into an output voltage of VPLUS/2 (+6 V in **Fig. 13-22**) and an input voltage of 0 V into an output voltage of VMINUS/2 (−6 V in **Fig. 13-22**).

